Revising Dynamic Programming Based Auto-Rebalancing Method and Comparing Performance with Urban Planning

修改並比較基於動態規劃與都市規劃之資產再平衡方法

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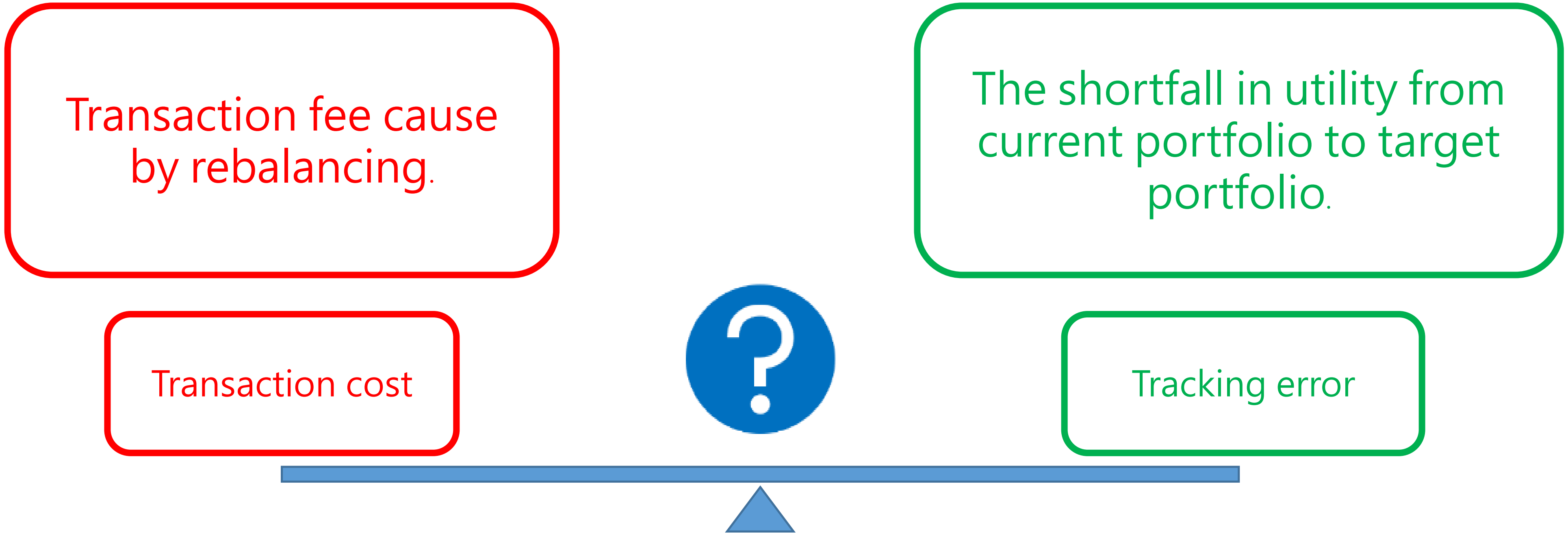
Chapter Ⅰ

Introduction

Introduction: What is Auto Rebalance?



Trade off between transaction cost and tracking error.

Chapter Ⅱ

Literature Review

Literature Review

- Leland(1999)
  - No-trade region
  - Obtain the boundary of no-trade region by solving PDE.
  - Do not adjust if the weight located in the no-trade region
  - Adjust the weight back to the boundary of no-trade region
- Donohue and Yip (2003)
  - Verify the model proposed by Leland(1999) with simulation data
  - Analyze the variables affecting the no-trade region.
    - Transaction cost and risk aversion – size
    - Correlation between the assets return - shape

Literature Review

- Helge Holden & Lars Holden(2013)
  - Discuss optimal rebalance of portfolios with transaction costs.
  - Proportional transaction cost: rebalance to the boundary of the no-trade region
  - Flat transaction cost: rebalance to an internal state of the no-trade region

Literature Review

- Walter Sun, Ayres Fan, Li-Wei Chen, Tom Schouwenaars and Marius A. Albota (2006)
  - Apply dynamic programming on optimal rebalancing strategies for portfolios.
  - Running value iteration algorithm.
  - Simulation data.
- 陳柏王 (2020)
  - Apply the model proposed by Sun et al.(2006) on strategic assets allocation(SAA) and mutual funds.
  - Running policy iteration algorithm instead.
  - Real data.
  - Soothe the curse of dimensionality by urban planning state selection method.

Contribution of the paper

- Modified dynamic programming based rebalancing method
    - Assets growth
    - Transaction costs
- Compare with different rebalancing method
    - Cost
    - Return
    - Sharpe ratio
- Verify the model in different risk aversion
- Analyze the effect of risk aversion and the size of state space

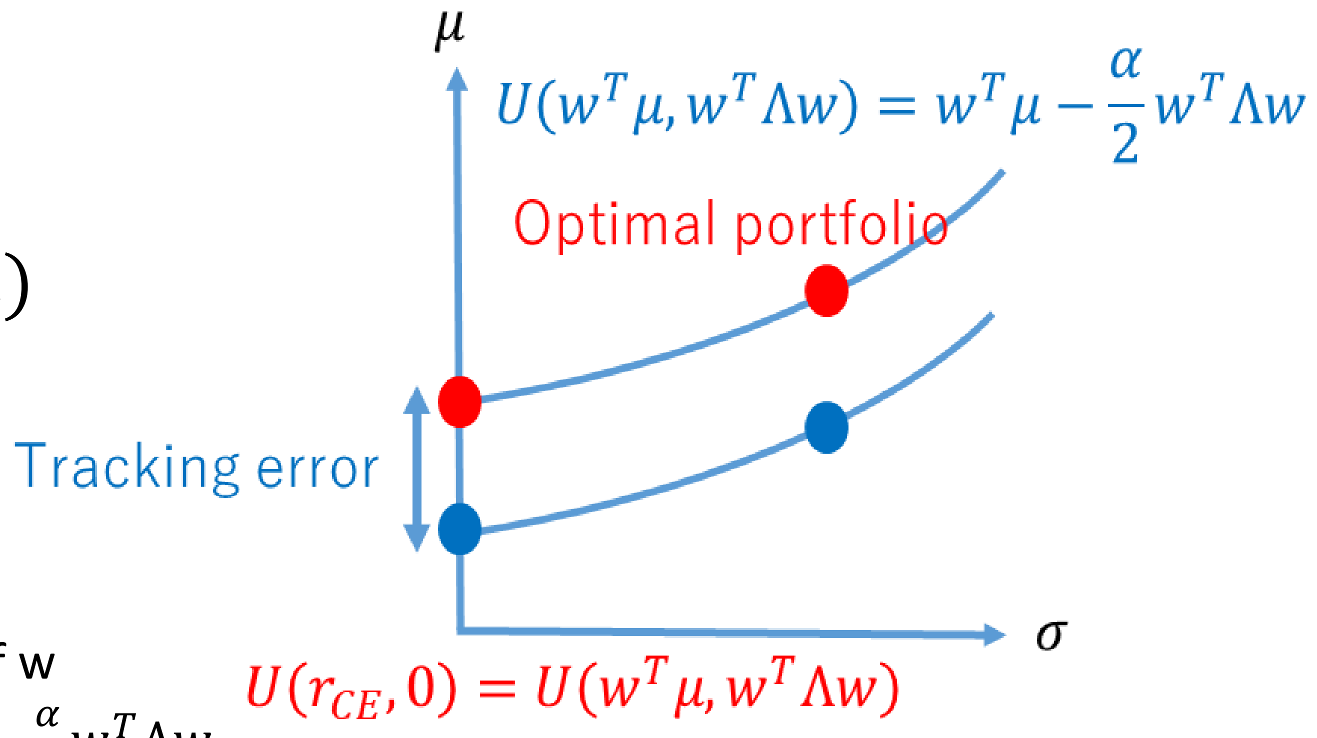
Chapter Ⅲ

Method

Cost

- $cost(w, a) = TE(a) + TC(w, a)$

- Tracking error(TE)
  - $TE(w) = r_{ce}(w^*) - r_{ce}(w)$
    - r_{ce}(w) is the certainty equivalent of w
    - $r_{ce}(w) = U(w^T\mu, w^T\Lambda w) = w^T\mu - \frac{\alpha}{2} w^T\Lambda w$
      - μ denotes expected return vector
      - Λ denotes covariance matrix
      - α denotes risk aversion coefficient
    - w* denotes the weight of optimal portfolio(target ratio)
      - $w^* = argmax_w U(w, \mu, \Lambda)$



Cost

- $cost(w, a) = TE(a) + TC(w, a)$

- Transaction cost(TC):
  - $TC(w, w') = \sum_{i=1}^{n} |w - w'|c \,, \begin{cases} c = 1.425‰\text{, if } w_i < w_i' \\ c = 4.425‰\text{, if } w_i > w_i' \end{cases}$

States construct, step1

- Calculate the lower and upper bound of each asset weight
  - $Lower\ Bound\ of\ asset\ i = \frac{w_i^*\ (1+\mu_i-k\sigma_i)}{w_i^*(1+\mu_i-k\sigma_i)+\sum_{j\neq i} w_i^*\left(1+\mu_j+k\sigma_j\right)}$
  - $Upper\ Bound\ of\ asset\ i = \frac{w_i^*\ (1+\mu_i+k\sigma_i)}{w_i^*(1+\mu_i+k\sigma_i)+\sum_{j\neq i} w_i^*\left(1+\mu_j-k\sigma_j\right)}$

States construct, step 2

- Construct the states with following constraints
  - The portfolio weight can be presented by a vector $w = [w_1, \dots, w_N]$
  - $\forall$ asset i, the weight w_i satisfy, $LB_i \leq w_i \leq UB_i$
  - The weight is discretized, so the possible weight of one asset may be, for example, {0.22,0.23,0.24,0.25,0.26,0.27,0.28}
  - Sum of w equals one
- For example, $N = 2$ and LB_i=0.49, $UB_i = 0.51$ for both assets.
  - w =[0.49,0.51], [0.50,0.50] or [0.51,0.49]

Transition probability

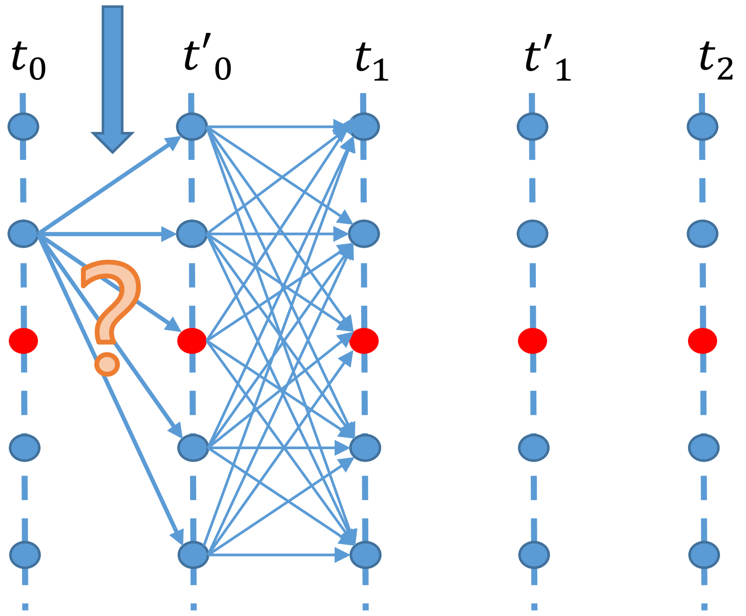
- Use Monte Carlo simulation to estimate the transition probability matrix

1. Estimate the correlation matrix Cor with historical data.
2. Use Cholesky decomposition $Cor = BB^T$
3. Randomly draw uncorrelated samples
   - $n = [n_1, \ldots, n_N]$, $\forall i = 1 \sim N$, $n_i \sim Normal(0,1)\ \ i.i.d$
4. Obtain $A = Bn$
   - $A = [a_1, \ldots, a_N]$, $Correlation\left(a_i, a_j\right) = \rho_{ij}\ \forall i, j = 1 \sim N$
5. Simulate the future price of the asset
   - $S_i(t) = S_i(t-1)e^{\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma a_i T}$

Value function

- $J_{i+1}(w) = min_a\{\underbrace{Cost(w,a)}_{\text{Cost induced by the policy}} + \underbrace{r\sum_{w'}[P(a,w')J_i(w')]}_{\text{E[future cost]}}\}$
  - w: state (weight)
  - a: policy (rebalance to the weight a)
  - Cost: tracking error and transaction cost induced by the policy a
  - r: discount factor
  - P(a,w'): the probability that state transit to w' after implementing policy a
  - $J_0(w) = 0$

Choose the policy with the lowest cost.



Modified value function

- Original:
  - $J_{n+1}(w) = min_a\{Cost(w,a) + r\sum_{w'}[P(a,w')J_n(w')]\}$
- Modified:
  - $J_{n+1}(w) = min_a\left\{Cost(w,a) + r(1 - TC(w,a\ ))\sum_{w'}[P(a,w')J_n(w')E(g|w')]\right\}$

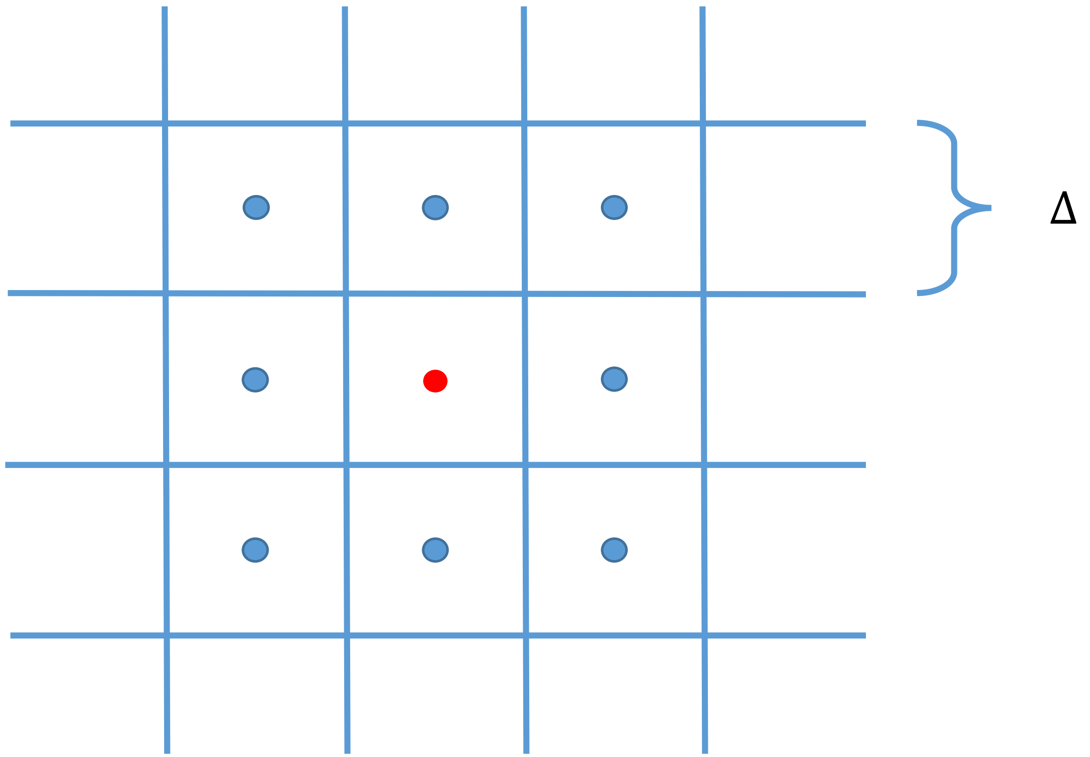
    - W: state (weight)
    - a: policy (rebalance to the weight a)
    - Cost: tracking error and transaction cost induced by the policy a
    - r : discount factor
    - P(a,w’): the probability that state transit to w’ after implementing policy a
    - g: assets growth
    - $J_0(w) = 0$

Policy iteration

- Initialize π_0
- $n \leftarrow 0$
- Repeat
  - Policy evaluation(repeat m times):
    - $J_n(w) \leftarrow Cost(w, \pi_{n-1}) + r\sum_{w'}[P(\pi_{n-1}, w')J_{n-1}(w')]$
  - Policy improvement
    - $\pi_{n+1}(w) \leftarrow argmin_a\{Cost(w, a) + r\sum_{w'}[P(a, w')J_n(w')]\}$
- Until π_{n+1} = π_n

States construct: Urban Planning

1. Divide the state space into several areas.

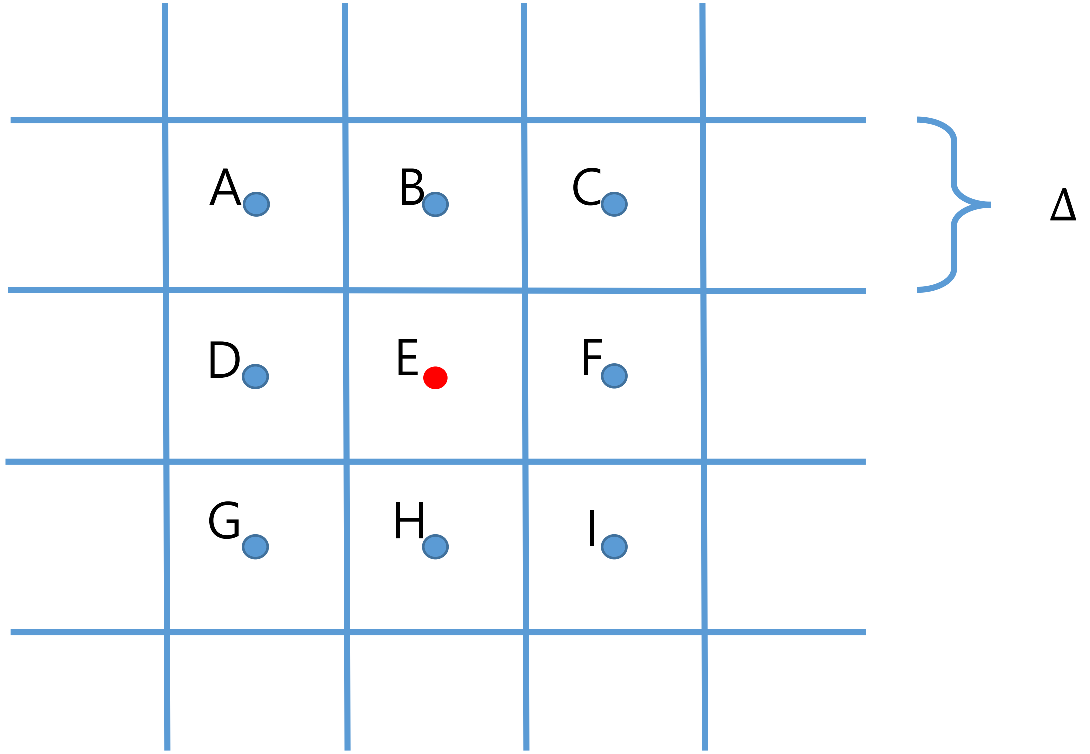


Red dot : target ratio

States construct: Urban Planning

2. Estimate the transition probability of area E.

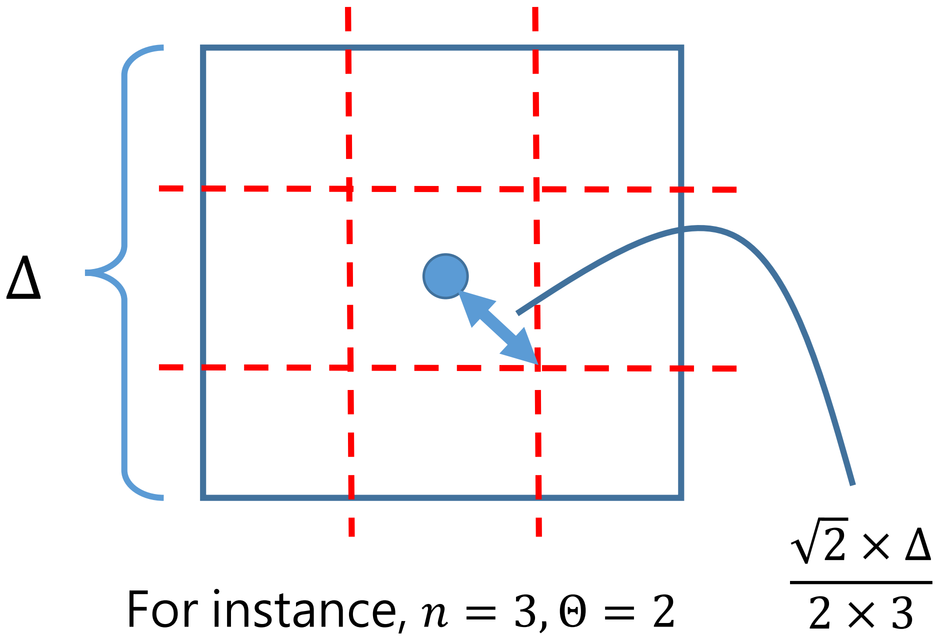
- $Prob(\mathrm{E} \rightarrow S)$, S could be A,B,C,D,E,F,G,H or I.



Red dot : target ratio

States construct: Urban Planning

3. Allocate the extra given grids ($grids\ added$) to the areas.



For instance, $n = 3, \Theta = 2$

Θ : number of assets (dimension)

$$Expected\ Error(i, n) = \frac{\sqrt{\Theta} \times \Delta}{2 \times n} \times Prob(E \rightarrow i)$$

$$Total\ Error = \sum_{area} Expected\ Error(area, allocate[area])$$

$$\min_{allocate} \ Total\ Error$$

$$s.t. \quad grids\ added - \sum_{area} allocate[area]^{\Theta} = 0$$

$for\ i\ in\ range\big(0, \#(area)\big):$
 $if\ allocate[i] < 1:$
  $allocate[i] = 1$
 $else:$
  $allocate[i] = floor(allocate[i])$

Chapter IV

Result

Data

- Daily price of stock market index from 1990 to 2019
  - SPX (Standard & Poor's 500 Index)
  - SXXP (STOXX Europe 600 Index)
  - NKY (Nikkei 225 Index)
  - SHCOMP (Shanghai Composite Index)
  - HSCEI (Hang Seng China Enterprises Index)
  - SENSEX (S&P Bombay Stock Exchange Sensitive Index)
  - TWSE (Taiwan Stock Exchange)
  - KOSPI (Korea Composite Stock Price Index)
  - MXSO (South East Asia)
  - MXMU (Emerging Markets Europe)
  - MXLA (Latin America)

We calculate the target ratio by maximizing utility function and remove the assets that the weight is lower than 5% from portfolio

Testing result

- 1 period = 21 trading days
- Estimate μ and Λ with the historical data from 1990~1999
- Test with simulation data(total 245 periods, about 20 years)

Parameter setting and target ratio

k=6	DP/mDP		UPDP/UPmDP		
	Δ	#(states)	Δ	G	#(states)
α=2	0.02	2634	0.03	2500	2398
α=3	0.02	1740	0.04	2000	1664
α=4	0.02	1284	0.03	1300	1215
α=5	0.02	1046	0.03	1000	1057
α=6	0.02	819	0.03	800	725

	SPX	SHCOMP	SENSEX	MXLA	Portfolio_mu	Portfolio_std
α=2	44.05%	26.94%	13.75%	15.26%	19.49%	16.72%
α=3	54.27%	19.86%	14.68%	11.19%	18.58%	14.29%
α=4	59.37%	16.31%	15.15%	9.16%	18.13%	13.34%
α=5	62.44%	14.19%	15.43%	7.94%	17.86%	12.87%
α=6	64.48%	12.77%	15.62%	7.13%	17.68%	12.61%

Testing result: $\alpha=2$

Estimation period: 1990~1999
Testing period: 20 years simulation data

	#(trading)	Total cost	Return(%)	Sharpe ratio
Dynamic programming(DP)	74.82	0.243317	4116.98	1.03970
Modified DP(mDP)	76.68	0.241467	4086.20	1.04447
Urban planning & DP(UPDP)	85.06	0.231515	4129.21	1.04297
Urban planning & mDP(UPmDP)	85.33	0.230816	4106.30	1.04743
Buy & Hold	0.00	33.014391	9112.83	0.84479
Periodic(1)	245.00	0.420946	2992.21	0.96429
Periodic(2)	122.00	0.347327	3402.23	0.98953
Periodic(3)	81.00	0.330041	3628.26	0.99954
Periodic(4)	61.00	0.337671	3747.44	1.00247
Periodic(5)	49.00	0.355821	3827.40	1.00279
Tolerance (1 %)	219.31	0.412269	3025.48	0.96695
Tolerance (2 %)	151.89	0.372331	3189.20	0.97944
Tolerance (3 %)	102.38	0.331656	3385.64	0.99333
Tolerance (4 %)	71.54	0.302456	3564.09	1.00503
Tolerance (5 %)	52.16	0.284595	3713.74	1.01411

Testing result: $\alpha=3$

Estimation period: 1990~1999
Testing period: 20 years simulation data

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	84.91	0.200828	3550.81	1.18154
mDP	83.57	0.200475	3558.52	1.18616
UPDP	92.19	0.187300	3636.71	1.19364
UPmDP	93.37	0.186393	3621.81	1.19569
Buy & Hold	0.00	42.278691	7394.01	0.934952
Periodic(1)	246.00	0.339500	2831.68	1.118889
Periodic(2)	112.00	0.280314	3148.99	1.143762
Periodic(3)	81.00	0.269041	3318.70	1.153413
Periodic(4)	61.00	0.278346	3408.62	1.156112
Periodic(5)	49.00	0.296835	3469.02	1.156237
Tolerance (1 %)	212.08	0.329143	2869.02	1.122626
Tolerance (2 %)	137.46	0.289392	3027.44	1.137834
Tolerance (3 %)	87.24	0.254090	3200.25	1.153295
Tolerance (4 %)	58.72	0.234182	3343.20	1.165040
Tolerance (5 %)	41.81	0.226985	3453.34	1.173328

Testing result: α=4

Estimation period: 1990~1999
Testing period: 20 years simulation data

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	82.13	0.177876	3356.30	1.25137
mDP	83.47	0.177779	3349.39	1.25112
UPDP	97.49	0.166956	3393.18	1.25828
UPmDP	97.86	0.166429	3384.21	1.25896
Buy & Hold	0.00	49.329684	6534.59	0.98299
Periodic(1)	245.00	0.296274	2741.64	1.19097
Periodic(2)	122.00	0.245703	3012.47	1.21512
Periodic(3)	81.00	0.238142	3154.91	1.22461
Periodic(4)	61.00	0.248717	3230.46	1.22743
Periodic(5)	49.00	0.267745	3281.15	1.22779
Tolerance (1 %)	205.80	0.284637	2782.36	1.19550
Tolerance (2 %)	126.77	0.245752	2937.20	1.21197
Tolerance (3 %)	77.45	0.216236	3094.03	1.22740
Tolerance (4 %)	51.01	0.204179	3213.61	1.23824
Tolerance (5 %)	35.89	0.205683	3304.65	1.24573

Testing result: $\alpha=5$

Estimation period: 1990~1999
Testing period: 20 years simulation data

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	77.52	0.168358	3246.20	1.28684
mDP	78.32	0.168457	3243.09	1.28655
UPDP	99.10	0.155927	3252.75	1.29005
UPmDP	100.10	0.155729	3250.85	1.29010
Buy & Hold	0.00	55.450306	6018.95	1.01272
Periodic(1)	245.00	0.269957	2684.47	1.22795
Periodic(2)	122.00	0.225278	2927.90	1.25143
Periodic(3)	81.00	0.220496	3054.60	1.26081
Periodic(4)	61.00	0.232325	3121.77	1.26383
Periodic(5)	49.00	0.252180	3166.80	1.26445
Tolerance (1 %)	200.76	0.257455	2727.87	1.23307
Tolerance (2 %)	118.95	0.220159	2879.99	1.25015
Tolerance (3 %)	70.88	0.195792	3024.67	1.26513
Tolerance (4 %)	46.08	0.189926	3129.35	1.27497
Tolerance (5 %)	32.14	0.198033	3206.83	1.28155

Testing result: α=6

Estimation period: 1990~1999
Testing period: 20 years simulation data

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	77.68	0.165704	3159.64	1.30374
mDP	81.97	0.164697	3120.13	1.30434
UPDP	98.77	0.150599	3155.98	1.30511
UPmDP	101.22	0.150260	3148.61	1.30514
Buy & Hold	0.00	61.102059	5675.18	1.03289
Periodic(1)	245.00	0.252370	2645.05	1.24859
Periodic(2)	122.00	0.212138	2870.54	1.27149
Periodic(3)	81.00	0.209684	2987.08	1.28079
Periodic(4)	61.00	0.222800	3048.80	1.28394
Periodic(5)	49.00	0.243691	3090.17	1.28480
Tolerance (1 %)	196.71	0.239270	2690.46	1.25413
Tolerance (2 %)	113.05	0.203694	2840.16	1.27147
Tolerance (3 %)	66.21	0.183874	2976.43	1.28595
Tolerance (4 %)	42.74	0.183386	3071.54	1.29499
Tolerance (5 %)	29.64	0.197536	3140.68	1.30077

Effect of α

Estimation period: 1990~1999
Testing period: 20 years simulation data

	#(trading)	Total cost	Return(%)	Sharpe ratio	Portfolio_mu	Portfolio_std
DP (α=2)	74.82	0.243317	4116.98	1.03970	19.49%	16.72%
DP (α=3)	84.91	0.200828	3550.81	1.18154	18.58%	14.29%
DP (α=4)	82.13	0.177876	3356.30	1.25137	18.13%	13.34%
DP (α=5)	77.52	0.168358	3246.20	1.28684	17.86%	12.87%
DP (α=6)	77.68	0.165704	3159.64	1.30374	17.68%	12.61%

- α ↑ -> Return↓ & Sharpe ratio↑ -> volatility↓↓

=> More risk averse

Test with another estimation period

- 1 period = 21 trading days
- Estimate μ and Λ with the historical data from 2000~2009
- Test with simulation data(total 121 periods, about 10 years)

Parameter setting and target ratio

k=6	DP/mDP		UPDP/UPmDP		
	Δ	#(states)	Δ	G	#(states)
α=2	0.02	348	0.04	300	349
α=3	0.02	387	0.04	350	393
α=4	0.03	979	0.05	1000	924
α=5	0.03	1120	0.05	1000	1015
α=6	0.03	1092	0.05	1000	990

	SHCOMP	HSCEI	SENSEX	MXLA	Portfolio_mu	Portfolio_std
α=2	0%	53.62%	21.65%	24.73%	17.55%	25.65%
α=3	0%	40.78%	30.64%	28.58%	16.58%	24.03%
α=4	11.64%	29.88%	30.90%	27.58%	14.87%	21.82%
α=5	17.78%	23.67%	31.36%	27.19%	13.93%	20.83%
α=6	21.88%	19.52%	31.67%	26.93%	13.30%	20.27%

Testing result: α=2

Estimation period: 2000~2009
Testing period: 10 years simulation data

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	24.48	0.015342	475.64	0.55355
mDP	26.79	0.015085	470.93	0.55501
UPDP	25.43	0.014962	474.60	0.55418
UPmDP	29.17	0.014656	472.64	0.55474
Buy & Hold	0.00	0.142176	520.28	0.53092
Periodic(1)	121.00	0.040873	445.98	0.54458
Periodic(2)	60.00	0.029802	456.19	0.54890
Periodic(3)	40.00	0.025655	460.52	0.55043
Periodic(4)	30.00	0.023555	463.89	0.55133
Periodic(5)	24.00	0.022327	464.89	0.55142
Tolerance (1 %)	100.52	0.039146	447.40	0.54526
Tolerance (2 %)	64.33	0.033123	452.47	0.54757
Tolerance (3 %)	40.05	0.027338	457.82	0.54983
Tolerance (4 %)	26.66	0.023430	462.00	0.55145
Tolerance (5 %)	18.82	0.020806	465.00	0.55233

Testing result: α=3

Estimation period: 2000~2009
Testing period: 10 years simulation data

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	32.49	0.017338	419.55	0.56863
mDP	32.49	0.017338	419.55	0.56863
UPDP	37.77	0.016723	420.01	0.56912
UPmDP	38.05	0.016697	419.12	0.56917
Buy & Hold	0.00	0.240269	465.17	0.54594
Periodic(1)	121.00	0.040928	397.83	0.55825
Periodic(2)	60.00	0.030295	406.68	0.56293
Periodic(3)	40.00	0.026576	410.44	0.56464
Periodic(4)	30.00	0.024925	412.84	0.56565
Periodic(5)	24.00	0.024161	414.20	0.56578
Tolerance (1 %)	103.96	0.039554	398.80	0.55882
Tolerance (2 %)	69.89	0.034188	402.77	0.56109
Tolerance (3 %)	44.15	0.028570	407.40	0.56360
Tolerance (4 %)	29.41	0.024718	410.94	0.56540
Tolerance (5 %)	20.79	0.022376	413.61	0.56648

Testing result: α=4

Estimation period: 2000~2009
Testing period: 10 years simulation data

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	30.09	0.019920	338.31	0.57149
mDP	30.82	0.019967	337.94	0.57136
UPDP	34.96	0.018319	342.92	0.57235
UPmDP	36.07	0.018143	341.88	0.57223
Buy & Hold	0.00	0.293690	396.02	0.54936
Periodic(1)	121.00	0.038946	323.54	0.56135
Periodic(2)	60.00	0.028940	330.24	0.56630
Periodic(3)	40.00	0.025534	333.32	0.56817
Periodic(4)	30.00	0.024099	335.32	0.56911
Periodic(5)	24.00	0.023598	336.94	0.56989
Tolerance (1 %)	106.99	0.037630	324.30	0.56195
Tolerance (2 %)	67.96	0.031505	328.12	0.56489
Tolerance (3 %)	41.11	0.026014	332.10	0.56769
Tolerance (4 %)	26.81	0.022752	335.08	0.56949
Tolerance (5 %)	18.84	0.021196	337.38	0.57062

Testing result: α=5

Estimation period: 2000~2009
Testing period: 10 years simulation data

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	33.10	0.020848	297.34	0.56280
mDP	33.10	0.020848	297.34	0.56280
UPDP	41.45	0.018208	301.00	0.56457
UPmDP	42.33	0.018218	300.73	0.56447
Buy & Hold	0.00	0.343583	355.89	0.54644
Periodic(1)	121.00	0.037200	286.80	0.55364
Periodic(2)	60.00	0.027918	292.56	0.55866
Periodic(3)	40.00	0.024936	295.28	0.56060
Periodic(4)	30.00	0.023837	297.05	0.56162
Periodic(5)	24.00	0.023631	298.50	0.56248
Tolerance (1 %)	108.53	0.036078	287.38	0.55420
Tolerance (2 %)	68.94	0.030195	290.68	0.55722
Tolerance (3 %)	40.99	0.024958	294.24	0.56016
Tolerance (4 %)	26.57	0.022202	296.86	0.56198
Tolerance (5 %)	18.58	0.021147	298.91	0.56322

Testing result: α=6

Estimation period: 2000~2009
Testing period: 10 years simulation data

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	34.62	0.021668	273.99	0.55327
mDP	34.62	0.021668	273.99	0.55327
UPDP	47.08	0.018432	276.24	0.55495
UPmDP	48.39	0.018337	275.60	0.55465
Buy & Hold	0.00	0.390294	329.13	0.54164
Periodic(1)	121.00	0.035873	264.07	0.54413
Periodic(2)	60.00	0.027229	269.24	0.54915
Periodic(3)	40.00	0.024657	271.73	0.55115
Periodic(4)	30.00	0.023900	273.34	0.55221
Periodic(5)	24.00	0.024006	274.66	0.55313
Tolerance (1 %)	108.66	0.034811	264.59	0.54468
Tolerance (2 %)	69.15	0.029248	267.54	0.54770
Tolerance (3 %)	41.04	0.024396	270.78	0.55069
Tolerance (4 %)	26.63	0.022064	273.25	0.55268
Tolerance (5 %)	18.63	0.021420	275.14	0.55389

Effect of α

Estimation period: 2000~2009
Testing period: 10 years simulation data

	#(trading)	Total cost	Return(%)	Sharpe ratio	Portfolio_mu	Portfolio_std
DP (α=2)	24.48	0.015342	475.64	0.55355	17.55%	25.65%
DP (α=3)	32.49	0.017338	419.55	0.56863	16.58%	24.03%
DP (α=4)	30.09	0.019920	338.31	0.57149	14.87%	21.82%
DP (α=5)	33.10	0.020848	297.34	0.56280	13.93%	20.83%
DP (α=6)	34.62	0.021668	273.99	0.55327	13.30%	20.27%

- α ↑ -> Return↓↓ & Sharpe ratio↑ (or ↓) -> volatility↓↓(or ↓)

=> More risk averse

Change the state space

- 1 period = 21 trading days
- Estimate μ and Λ with the historical data from 1990~1999
- Test with simulation data(total 252 periods, about 20 years)
- Set k=3 (the parameter of upper bound and lower bound for assets weight)

Parameter setting and target ratio

k=3	DP/mDP		UPDP/UPmDP		
	Δ	#(states)	Δ	G	#(states)
α=2	0.01	2513	0.015	2500	2164
α=4	0.01	1224	0.015	1500	1227
α=6	0.01	771	0.015	1000	779

	SPX	SHCOMP	SENSEX	MXLA	Portfolio_mu	Portfolio_std
α=2	44.05%	26.94%	13.75%	15.26%	19.49%	16.72%
α=4	59.37%	16.31%	15.15%	9.16%	18.13%	13.34%
α=6	64.48%	12.77%	15.62%	7.13%	17.68%	12.61%

α=2,上下界3倍sigma

Estimation period: 1990~1999
Testing period: 20 years simulation data

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	80.82	0.236974	4173.71	1.04072
mDP	82.13	0.232759	4123.35	1.04652
UPDP	81.60	0.237998	4157.72	1.03932
UPmDP	83.07	0.235778	4136.44	1.04511

α=2,上下界6倍sigma

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	74.82	0.243317	4116.98	1.03970
mDP	76.68	0.241467	4086.20	1.04447
UPDP	85.06	0.231515	4129.21	1.04297
UPmDP	85.33	0.230816	4106.30	1.04743

α=4,上下界3倍sigma

Estimation period: 1990~1999
Testing period: 20 years simulation data

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	92.56	0.172542	3406.31	1.25651
mDP	94.19	0.171490	3401.60	1.25749
UPDP	95.15	0.168498	3394.92	1.25671
UPmDP	97.62	0.167185	3386.99	1.25730

α=4,上下界6倍sigma

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	82.13	0.177876	3356.30	1.25137
mDP	83.47	0.177779	3349.39	1.25112
UPDP	97.49	0.166956	3393.18	1.25828
UPmDP	97.86	0.166429	3384.21	1.25896

α=6,上下界3倍sigma

Estimation period: 1990~1999
Testing period: 20 years simulation data

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	106.16	0.152787	3144.65	1.30396
mDP	106.68	0.152435	3141.72	1.30400
UPDP	111.84	0.147437	3142.78	1.30466
UPmDP	114.28	0.147134	3136.27	1.30458

α=6,上下界6倍sigma

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	77.68	0.165704	3159.64	1.30374
mDP	81.97	0.164697	3120.13	1.30434
UPDP	98.77	0.150599	3155.98	1.30511
UPmDP	101.22	0.150260	3148.61	1.30514

Test with real market data

- 1 period = 21 trading days
- Estimate μ and Λ with the historical data from 1990~1999
- Test from 2000~2019 (total 245 periods)
- Given α=4, target ratio:
  - SPX 59.37%；SHCOMP 16.31%；SENSEX 15.15%；MXLA 9.16%

$\alpha=4$

Estimation period: 1990~1999
Testing period: 2000~2019

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	47	0.020243	233.23	0.35456
mDP	47	0.019686	227.39	0.34989
UPDP	77	0.018260	225.09	0.34823
UPmDP	74	0.018008	225.47	0.34873
Buy & Hold	0	0.212349	191.58	0.30332
Periodic(1)	245	0.027886	201.65	0.32843
Periodic(2)	122	0.021830	205.95	0.33325
Periodic(3)	81	0.019300	208.15	0.33514
Periodic(4)	61	0.019669	207.57	0.33488
Periodic(5)	49	0.018036	214.37	0.34121
Tolerance (1 %)	135	0.022748	203.29	0.32990
Tolerance (2 %)	60	0.016618	205.12	0.33077
Tolerance (3 %)	34	0.016000	207.39	0.33469
Tolerance (4 %)	25	0.018578	219.80	0.34474
Tolerance (5 %)	15	0.023283	220.56	0.34632

Test with real market data

- 1 period = 21 trading days
- Estimate μ and Λ with the historical data from 2000~2009
- Test from 2010~2019 (total 121 periods)
- Given α=5, target ratio:
  - SHCOMP 17.78% ; HSCEI 23.67% ; SENSEX 31.36% ; MXLA 27.19%

$\alpha=5$

Estimation period: 2000~2009
Testing period: 2009~2019

	#(trading)	Total cost	Return(%)	Sharpe ratio
DP	13	0.005860	15.87	0.09458
mDP	13	0.005860	15.87	0.09458
UPDP	20	0.004398	15.97	0.09478
UPmDP	21	0.004427	15.94	0.09465
Buy & Hold	0	0.048999	21.68	0.13067
Periodic(1)	121	0.009735	13.56	0.08107
Periodic(2)	60	0.007478	14.67	0.08732
Periodic(3)	40	0.006448	14.72	0.08734
Periodic(4)	30	0.005792	13.55	0.08111
Periodic(5)	24	0.004828	13.30	0.07961
Tolerance (1 %)	68	0.007939	13.80	0.08248
Tolerance (2 %)	29	0.005342	13.37	0.08010
Tolerance (3 %)	14	0.004544	13.94	0.08327
Tolerance (4 %)	9	0.003894	12.98	0.07773
Tolerance (5 %)	7	0.004255	13.46	0.08044

Chapter V

Conclusion

Conclusion

1. 使用simulation data進行測試，平均來看動態平衡法不論在total cost、return或Sharpe ratio方面都有很好的表現，說明當我們對市場的機率模型估計得夠精確，動態平衡法能為投資人提供良好且穩定的資產再平衡策略。
2. 使用real data進行測試，動態平衡法的表現並非最佳。
   - Real data可視為其中一次simulation的結果，動態平衡法並非在每次simulation中都會有最佳的表現。但是動態平衡法的表現夠穩定，因此進行多次simulation平均來看動態平衡法會是最佳的策略。
   - 實際的市場狀況可能與我們預估的不同
3. 當α越大，累積return越小。代表投資人選擇較低的風險與報酬，反映出投資人風險趨避的情況。

Conclusion

4. 進行動態規劃時考慮資產成長，能提升動態平衡法的表現(mDP>DP, UPmDP>UPDP)。當α越小，投資組合的波動度越大，考慮資產成長對動態平衡法表現的提升就越顯著。
5. 在狀態空間小、 α小的情況下，使用都市規劃的動態平衡法會輸給未使用都市規劃的動態平衡法
   - 當狀態空間過小，動態規劃求解的過程中可能會遺漏重要的訊息導致都市規劃的表現變差。
6. 在狀態空間大的情況下使用都市規劃法，表現會優於未使用都市規劃的動態平衡法(不論狀態空間大小)。
   - 較大的狀態空間使都市規劃法能夠更明確的劃分各個區域
7. 在有限的狀態數下，都市規劃能提升動態平衡法的效果(UPDP>DP, UPmDP>mDP)。
   - 當狀態空間變大時，未使用都市規劃的動態平衡法表現明顯下降

Thank you for listening